Visualizing Engineering Mathematics in Action!

Michael J. Gordon

Dept. of Chemical Engineering, University of California - Santa Barbara

The PROBLEM: Learning, appreciating, and using advanced mathematics to solve problems is tough!

Advanced mathematics (vector calculus, ordinary/partial differential equations, Fourier analysis, and numerical methods) in various discipline-specific contexts (e.g., fluid mechanics, heat/mass transport, materials physics, circuits, and process control) is a cornerstone of engineering. However, undergraduate students are routinely inexperienced and uncomfortable in this domain because our current instructional approach to mathematics is rather archaic, i.e., teaching math just for the sake of teaching math, rather than through the use of applications and visualization.

The SOLUTION: Connect analytical and numerical methods with real world examples through *visualization*

Make complex engineering math more approachable to students by teaching math in an application-oriented manner using *relevant and timely* examples with real-time, hands-on *visualization* and *animation* of solutions by students in a computer lab-based setting.

Educational Objectives

• Increase student understanding of and comfort with vector calculus, ODE/PDEs, variable transforms, and numerical methods to solve engineering problems.

• Help students make important connections between analytical solutions and real world problems using advanced mathematical tools they will encounter in industry and research.

• Teach students how to use Mathematica and MATLAB to solve, manipulate, and *visualize* solutions to complex engineering problems involving fluid flow, heat/mass transport, materials, process control, and reaction kinetics.

Learning Activities and Course Development

ChE132a: Analytical Methods of Chemical Engineering

Junior level course with ***Mathematica***

3 x lectures + 2 hrs computer lab / week

Ex #1: Analyze the vibrations of a cylindrical drum

We must solve: $\dfrac{\partial^2 u}{\partial t^2} = a^2 \nabla^2 u$

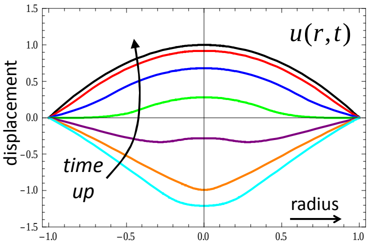
By hand, we find:

$$u(r,t) = \sum_{m=1}^{\infty} b_m J_0(\lambda_m r)\cos(a\lambda_m t)$$

$$b_m = \frac{2}{J_1^2(\lambda_m)}\int_0^1 r(1-r^2)J_0(\lambda_m r)\,dr$$

What does this mean?

Visualize and *animate* PDE solution to understand motion

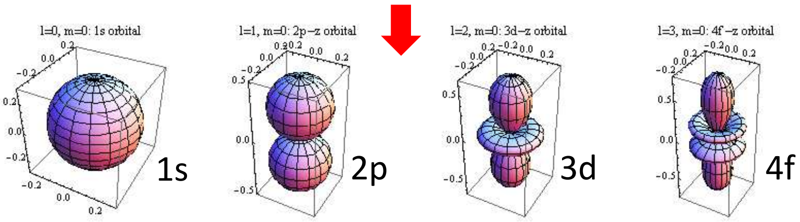


Ex #2: Where are the electrons in an atom?

We must solve Schrodinger's equation in spherical coordinates:

$$-\frac{\hbar^2}{2\mu}\frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\Psi}{\partial\phi^2}\right]+U(r)\Psi(r,\theta,\phi)=E\Psi(r,\theta,\phi)$$

We solve this by hand over 5 lectures to demonstrate separation of variables, coordinate transforms, and power series… **but**, *what do the answers mean?*



Connect mathematical methods for PDE solutions with Physical Chemistry and discover why orbitals look the way they do!

ChE132b: Numerical Methods of Chemical Engineering

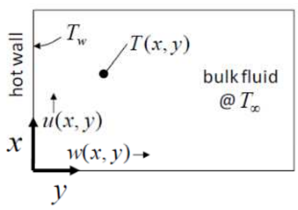
Senior level course with ***MATLAB***

3 x lectures + 2 hrs computer lab / week

Ex #3: Natural convection near a hot vertical surface (e.g., windows, cooling fins, pipe insulation…)

Temperature gradients cause buoyancy differences & fluid flow

Model geometry



We must numerically solve coupled non-linear ODEs for fluid flow & heat transfer

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\frac{T_w-T_\infty}{T_\infty}\theta$$

$$u\frac{\partial\theta}{\partial x}+w\frac{\partial\theta}{\partial y}=\alpha\frac{\partial^2\theta}{\partial y^2}$$

MATLAB

What does the flow profile look like?