How to simplify a signals-and-systems course by using operators

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The need

The core ideas of signals and systems—poles, modes, linearity, feedback, and control—are a fundamental contribution of electrical engineering and computer science (EECS) to our understanding and mastery of the world, perhaps as important as the ideas of mechanics.

However, for two related reasons, the beauty of and power of the signals-and-systems ideas are not appreciated by most students:

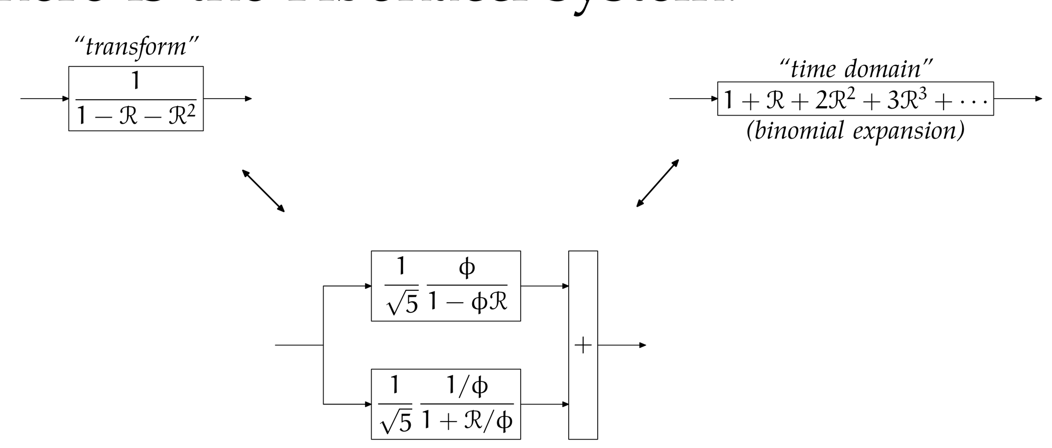
1. **Students oriented toward computation see signals and systems as a "circuits" course** to be avoided except under duress.

2. The ideas are taught primarily through continuous-time systems, whose analysis requires the complicated machinery of analysis including the setting up, solving, and understanding of differential equations. Like gauze over a camera lens, this **mathematical complexity obscures the core EECS ideas.**

My solution

The innovation described here, in teaching cutting-edge engineering knowledge, thins the gauze and expands the audience for these ideas by making discrete-time systems the heart of the course.

This change is enabled by describing discrete-time systems using shift (delay) operators. The actors in the course become just two: signals and systems, which are represented as ratios of polynomials in the shift operators. Students' can then use their knowledge of polynomials from school mathematics to analyze and design systems. Even the difficult concept of transform simplifies almost into nonexistence. For example, here is the Fibonacci system:



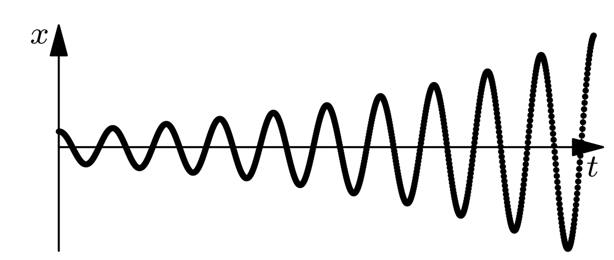
Developmental history

I developed this operator approach in 2007 when I co-taught "6.003. Signals and systems" at MIT with my colleague Dennis Freeman. Our unease with the usual course started as a small thread. We were frustrated by the following notation for a convolution of two signals:

$$f[n-1] \star g[n-1] = h[n-2].$$

Pulling on that one thread unraveled the whole course, and led to its rebuilding using the operator approach.

Example problems



Use the forward-Euler approximation to make a discrete-time simulation of an undamped spring–mass system. Explain why the simulated position versus time (shown in the figure) so badly represents the behavior of an actual spring.

A system takes the unit impulse and turns it into an intermediate signal. If the system had been fed that intermediate signal, it would have produced the unit step function. What is the intermediate signal?

Execution

I teach this approach at Olin College of Engineering. At MIT it has become the standard approach in the first-semester EECS core course: "6.01. Introduction to Electrical Engineering and Computer Science (EECS)." It is also frequently the approach in the junior-level signals-and-systems course (6.003).

Discrete-time systems using operators forms the first one-third of a course on signals and systems. That module introduces most of the core ideas of the course: poles, zeros, modes, linearity, feedback, and control. The remaining two modules are continuous-time systems and the Fourier transform.

Major issues

First, the approach requires mathematics that we expect students to understand but that they mostly do not:

1. linear algebra

2. the complex plane

Second, how can the approach be generalized to continuous-time systems? My current best guess uses integration operators (instead of the Laplace transform). However, they have not made continuous-time systems as intuitive as the shift operators have made discrete-time systems. A related problem is the lack of a smooth connection to the Fourier transform.

Spinoffs

An expected spinoff is a **short textbook** based on the course notes and roughly entitled *A Student's Guide to Discrete-Time Signals and Systems*.

Acknowledgments

My great debt is to **Dennis Freeman**, with whom I taught the first course using this approach.